Exercise 52

For the following exercises, which of the tables could represent a linear function? For each that could be linear, find a linear equation that models the data.

x	2	4	6	8
$f(x)$	−4	16	36	56

Solution

This table represents a linear function because as x increases by 2, $f(x)$ increases by 20. Two points on this line are
$$(2, -4) \text{ and } (4, 16).$$
The general equation for a line is
$$y = mx + b.$$
The first point says that when $x = 2$, $y = -4$.
$$-4 = m(2) + b$$
The second point says that when $x = 4$, $y = 16$.
$$16 = m(4) + b$$
This is a system of two equations with two unknowns that can be solved.
$$\begin{cases} 2m + b = -4 \\ 4m + b = 16 \end{cases}$$
Subtract the respective sides of these equations to eliminate b.
$$2m - 4m = -4 - 16 \quad \rightarrow \quad -2m = -20 \quad \rightarrow \quad m = 10$$
Multiply both sides of the first equation by -2
$$\begin{cases} -4m - 2b = 8 \\ 4m + b = 16 \end{cases}$$
and then add the respective sides to eliminate m.
$$-2b + b = 8 + 16 \quad \rightarrow \quad -b = 24 \quad \rightarrow \quad b = -24$$
Now that m and b have been solved for, the line is known.
$$y = 10x - 24$$